Gas Power Cycles

Specific Heats for Ideal Gases

Internal energy and enthalpy changes, specific heats constant: \( \Delta u = c_v \Delta T \quad \Delta h = c_p \Delta T \)

Air, 300 K: \( R = 0.2870 \, \text{kJ/kg-K} \quad c_p = 1.005 \, \text{kJ/kg-K} \quad c_v = 0.718 \, \text{kJ/kg-K} \quad k = 1.4 \)

Isentropic Processes of Ideal Gases with constant specific heats

\[
\frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{(k-1)}
\]

\[
\frac{T_b}{T_a} = \left( \frac{P_b}{P_a} \right)^{(\frac{k-1}{k})}
\]

\[
\frac{P_b}{P_a} = \left( \frac{v_1}{v_2} \right)^k
\]

First Law Simplifications

Closed system, heat transfer at constant volume: \( q_{ab} = u_b - u_a \)

Closed system, heat transfer at constant pressure: \( q_{ab} = h_b - h_a \)

Closed system, isentropic compression or expansion: \( w_{ab} = u_a - u_b \)

(where \( a \) and \( b \) represent two states in the cycle.)

Open system, heat addition or rejection: \( q = h_e - h_i \)

Open system, isentropic compression or expansion: \( w = h_i - h_e \)

(where \( i \) and \( e \) represent inlet and exit states of a device in the cycle.)

\[
r = \frac{v_{\text{max}}}{v_{\text{min}}}
\]

\[
r_p = \frac{P_{\text{max}}}{P_{\text{min}}}
\]

\[
\text{MEP} = \frac{w_{\text{net}}}{v_{\text{max}} - v_{\text{min}}}
\]

\[
\eta_C = \frac{w_r}{w_a}
\]

\[
\eta_T = \frac{w_r}{w_s}
\]

Otto

Ideal cycle: \( \eta_{th} = 1 - \frac{1}{r^{k-1}} \)

1-2 Isentropic compression
2-3 Constant volume heat addition
3-4 Isentropic expansion
4-1 Constant volume heat rejection

Diesel

\[
r_c = \frac{v_3}{v_2}
\]

Ideal cycle: \( \eta_{th} = 1 - \frac{1}{r^{k-1} \left[ \frac{r^k - 1}{k(r_c - 1)} \right]} \)

1-2 Isentropic compression
2-3 Constant pressure heat addition
3-4 Isentropic expansion
4-1 Constant volume heat rejection
Brayton

Ideal cycle: \( \eta_{th} = 1 - \frac{1}{(r_p)^{k-1}/e} \)

1-2 Isentropic compression
2-3 Constant pressure heat addition
3-4 Isentropic expansion
4-1 Constant pressure heat rejection

Ideal cycle with regeneration: \( \eta_{th} = 1 - \left( \frac{T_3}{T_1} \right) (r_p)^{(k-1)/k} \quad \epsilon = \frac{q_{regen, act}}{q_{regen, ideal}} \)

with reheat (2-stage turbine): \( q_{in} = q_{primary} + q_{reheat} \quad \text{and} \quad w_T = w_{turb I} + w_{turb II} \)

\( w_{T, max} \) occurs when \( r_{p,turb I} = r_{p,turb II} \).

with intercooling (2-stage compressor): \( w_C = w_{comp I} + w_{comp II} \)

\( w_{C, min} \) occurs when \( r_{p,comp I} = r_{p,comp II} \).

**Vapor Power Cycles**

Quality of a saturated mixture

\[ x = \frac{m_{vap}}{m_{tot}} \quad y = y_f + xy_{fg} = (1 - x)y_f + xy_g \quad \text{where} \quad y \text{ is any intensive property in the tables} \]

Compressed liquid approximation: \( y \approx y_f @ T \)

Pump work approximation: \( v_1 \approx v_2 \quad \text{and} \quad |w_p| \approx v_1(P_2 - P_1) \)

First Law Simplifications

Open system, heat addition or rejection: \( q = h_e - h_i \)

Open system, isentropic compression or expansion: \( w = h_i - h_e \)

(\( where \ i \ and \ e \ represent \ inlet \ and \ exit \ states \ of \ a \ device \ in \ the \ cycle. \))

\[ \eta_P = \frac{w_p}{w_a} \quad \text{and} \quad \eta_T = \frac{w_T}{w_a} \]

**Rankine**

\( \eta_{th} = \frac{w_{net}}{q_{in}} \)

Ideal cycle:

1-2 Isentropic compression
2-3 Constant pressure heat addition
3-4 Isentropic expansion
4-1 Constant pressure heat rejection

Ideal cycle with reheat (2-stage turbine): \( q_{in} = q_{primary} + q_{reheat} \quad \text{and} \quad w_T = w_{turb I} + w_{turb II} \)

with regeneration: \( w_T = w_{turb I} + (1 - y)w_{turb II} \quad \text{where} \ y \text{ is the fraction of steam extracted} \)

Fraction of steam extracted in an open feedwater heater: \( y = \frac{h_3 - h_2}{h_a - h_2} \)

Steady-state feedwater heater energy balance: \[ \sum m_{in} h_{in} = \sum m_{out} h_{out} \]
Vapor Compression Refrigeration Cycles

\[ \text{COP}_R = \frac{q_L}{w_{in}} = \frac{h_1 - h_4}{h_2 - h_1} \]

\[ \text{COP}_{HP} = \frac{q_H}{w_{in}} = \frac{h_2 - h_3}{h_2 - h_1} \]

\[ \text{COP}_{\text{Carnot}} = \frac{1}{T_H/T_L - 1} \]

Ideal cycle:
1-2 Isentropic compression
2-3 Constant pressure heat rejection
3-4 Throttling with an expansion valve
4-1 Constant pressure heat addition

Ideal Gases and Gas Mixtures

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\( \Delta h = c_p \Delta T \)

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Isentropic Processes of Ideal Gases with constant specific heats

\[ \frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{(k-1)} \]
\[ \frac{P_2}{P_1} = \left( \frac{v_1}{v_2} \right)^{(k-1)} \]
\[ \frac{P_2}{T_1} = \left( \frac{v_1}{v_2} \right)^k \]

Mass fraction: \( m_{f_i} = \frac{m_i}{m} \)
Mole fraction: \( y_i = \frac{N_i}{N_m} \)

Apparent molar mass: \( M_m = \frac{m}{N_m} \)
Apparent gas constant: \( R_m = \frac{R}{M_m} \)

\[ m_{f_i} = \frac{N_i}{N_m} M_i = y_i \frac{M_i}{M_m} \]

Additive pressures (Dalton’s Law): \( P_m = \sum P_i(T_m, V_m) \)
Additive volumes (Amagat’s Law): \( V_m = \sum V_i(T_m, P_m) \)
Ideal Gases: \( P_i/P_m = V_i/V_m = N_i/N_m \)

Psychrometrics, Heating and Cooling Processes

\[ \omega = \frac{m_a}{m_a} = \frac{0.622 P_a}{P - P_a} \]
\[ \phi = \frac{m_v}{m_v} = \frac{P_v}{P_g} \]

\[ h_{\text{atmospheric air}} = h_a + \omega h_g \]

with subscripts \( a \) – dry air, \( v \) – water vapor or vapor pressure, \( g \) – saturation pressure

Mass and Energy Balances, Steady State Processes (negligible \( \Delta KE, \Delta PE \))

\[ \sum \dot{m}_{in} = \sum \dot{m}_{out} \]
\[ \sum \dot{m}_{in}(h_{in}) + \dot{Q}_{\text{net in}} = \sum \dot{m}_{out}(h_{out}) + \dot{W}_{\text{net out}} \]